

Lecture 02: Language Modeling



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How many words?

- I do mainly business data processing
 - Fragments
 - Filled pauses
- Are **cat** and **cats** the same word?
- Some terminology
 - **Lemma**: a set of lexical forms having the same stem, major part of speech, and rough word sense
 - Cat and cats = **same lemma**
 - **Wordform**: the full inflected surface form.
 - Cat and cats = **different wordforms**

Moving toward language

Probability and part of speech tags

- What's the probability of a random word (from a random dictionary page) being a verb?
- How to compute each of these:
 - All words = just count all the words in the dictionary
 - # of ways to get a verb: number of words which are verbs!
 - If a dictionary has 50,000 entries, and 10,000 are verbs
 - **$P(V)$ is $10000/50000 = 0.2$**

How many words?

- **Token** is an **individual occurrence** of a linguistic unit in speech or words in a text.
- **Type** is the number of **distinct** linguistic unit in speech or words in a text.
- Thus, the sentence "a good food is a food that you like" contains **nine** tokens, but only **seven** types, as "a" and "food" are repeated.
- Switchboard-1 Telephone Speech Corpus (SWBD):
 - ~20,000 wordform types,
 - 2.4 million wordform tokens
- Brown et al (1992) large corpus
 - 293,181 wordform types
 - 583 million wordform tokens
- Let N = number of tokens, V = vocabulary = number of types
- General wisdom: $V > O(\sqrt{N})$

Language Modeling

- We want to compute $P(w_1, w_2, w_3, w_4, w_5 \dots w_n)$, the probability of a sequence.
- Alternatively we want to compute $P(w_5 | w_1, w_2, w_3, w_4)$: the probability of a word given some previous words.
- The model that computes $P(W)$ or $P(w_n | w_1, w_2 \dots w_{n-1})$ is called the **language model**.
- A better term for this would be “The Grammar”.
- But “Language model” or LM is standard.

Computing $P(W)$

- How to compute this joint probability:
 - $P(\text{"the", "other", "day", "I", "was", "walking", "along", "and", "saw", "a", "lizard"})$
- Note: let's rely on the Chain Rule of Probability

The Chain Rule of Probability

- Recall the definition of conditional probabilities

$$P(A | B) = \frac{P(A \wedge B)}{P(B)}$$

- Rewriting:

$$P(A \wedge B) = P(A | B)P(B)$$

- More generally

$$P(A, B, C, D) = P(A) P(B | A) P(C | A, B) P(D | A, B, C)$$

- In general

$$P(x_1, x_2, x_3, \dots, x_n) = P(x_1) P(x_2 | x_1) P(x_3 | x_1, x_2) \dots P(x_n | x_1 \dots x_{n-1})$$

The Chain Rule Applied to joint probability of words in sentence

$$\begin{aligned} P(w_1^n) &= P(w_1)P(w_2|w_1)P(w_3|w_1^2)\dots P(w_n|w_1^{n-1}) \\ &= \prod_{k=1}^n P(w_k|w_1^{k-1}) \end{aligned}$$

- $P(\text{"the big red dog was"}) = P(\text{the}) * P(\text{big}|\text{the}) * P(\text{red}|\text{the big}) * P(\text{dog}|\text{the big red}) * P(\text{was}|\text{the big red dog})$

Unfortunately

- There are a lot of possible sentences
- We'll never be able to get enough data to compute the statistics for those long prefixes:
P(lizard | the,other,day,I,was,walking,along,and,saw,a)

Markov Assumption

- Make the simplifying assumption
 - $P(\text{lizard} | \text{the, other, day, I, was, walking, along, and, saw, a}) = P(\text{lizard} | \text{a})$
- Or maybe
 - $P(\text{lizard} | \text{the, other, day, I, was, walking, along, and, saw, a}) = P(\text{lizard} | \text{saw, a})$

An example

- `<s> I am Sam </s>`
- `<s> Sam I am </s>`
- `<s> I do not like green eggs and ham </s>`

$$P(\text{I} | \text{<s>}) = \frac{2}{3} = .66$$

$$P(\text{</s>} | \text{Sam}) = \frac{1}{2} = 0.5$$

$$P(\text{do} | \text{I}) = \frac{1}{3} = .33$$

$$P(\text{Sam} | \text{<s>}) = \frac{1}{3} = .33$$

$$P(\text{<s>} | \text{Sam}) = \frac{1}{2} = 0.5$$

$$P(\text{am} | \text{I}) = \frac{2}{3} = .33$$

$$P(\text{Sam} | \text{am}) = \frac{1}{2} = .5$$

Maximum Likelihood Estimates

- The maximum likelihood estimate of some parameter of a model M from a training set T
 - Is the estimate
 - that maximizes the likelihood of the training set T given the model M
- Suppose the word Chinese occurs 400 times in a corpus of a million words (Brown corpus)
- What is the probability that a random word from some other text will be “Chinese”
- MLE estimate is $400/1000000 = .004$
 - This may be a bad estimate for some other corpus
- But it is the **estimate** that makes it **most likely** that “Chinese” will occur 400 times in a million word corpus.

More examples: Berkeley Restaurant Project sentences

- mid priced thai food is what i'm looking for
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

- $P(\langle s \rangle \text{ I want english food } \langle /s \rangle) =$
 $p(i | \langle s \rangle) \times p(\text{want} | i) \times p(\text{english} | \text{want}) \times p(\text{food} | \text{english}) \times$
 $p(\langle /s \rangle | \text{food})$
 $= 0.24 \times 0.33 \times 0.0011 \times 0.5 \times 0.68$
 $= 0.000031$

What kinds of knowledge?

- $P(\text{english} | \text{want}) = 0.0011$
- $P(\text{chinese} | \text{want}) = 0.0065$
- $P(\text{to} | \text{want}) = 0.66$
- $P(\text{eat} | \text{to}) = 0.28$
- $P(\text{food} | \text{to}) = 0$
- $P(\text{want} | \text{spend}) = 0$
- $P(i | \langle s \rangle) = 0.25$

The Shannon Visualization Method

- Generate random sentences
- Choose a random bigram $\langle s \rangle, w$ according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose $\langle /s \rangle$
- Then string the words together

- $\langle s \rangle$ I

I want

want to

to eat

eat Chinese

Chinese food

food $\langle /s \rangle$

Unigram	<ul style="list-style-type: none"> • To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have • Every enter now severally so, let • Hill he late speaks; or! a more to leg less first you enter • Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like
Bigram	<ul style="list-style-type: none"> • What means, sir. I confess she? then all sorts, he is trim, captain. • Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. • What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman? • Enter Menenius, if it so many good direction found'st thou art a strong upon command of fear not a liberal largess given away, Falstaff! Exeunt
Trigram	<ul style="list-style-type: none"> • Sweet prince, Falstaff shall die. Harry of Monmouth's grave. • This shall forbid it should be branded, if renown made it empty. • Indeed the duke; and had a very good friend. • Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.
Quadrigram	<ul style="list-style-type: none"> • King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; • Will you not tell me who I am? • It cannot be but so. • Indeed the short and the long. Marry, 'tis a noble Lepidus.

Shakespeare as corpus

- $N=884,647$ tokens, $V=29,066$
- Shakespeare produced 300,000 bigram types out of $V^2= 844$ million possible bigrams: so, 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it *is* Shakespeare